





Milestone about "Fuzzy control"

Thierry Marie GUERRA

LAMIH, UMR CNRS 8201, Université de Valenciennes et du Hainaut-Cambrésis Le Mont Houy, 59313 Valenciennes Cedex 9, France Tel : (+33) 03 27 51 13 37 e-mail: guerra@univ-valenciennes.fr





Where I come from...



70 Researchers, 20 Administrative staff, 70 PhD and Postdocs

Identity | Human-centered Safety, Mobility and Interacting Systems

"Fuzzy milestone" As a matter of introduction

"Fuzzy theory is wrong, wrong, and pernicious. What we need is more logical thinking, not less. The danger of fuzzy logic is that it will encourage the sort of imprecise thinking that has brought us so much trouble. **Fuzzy logic is the cocaine of science**." Professor **William Kahan** UC Berkeley

"Fuzzification' is a kind of scientific permissiveness. It tends to result in socially appealing slogans unaccompanied by the discipline of hard scientific work and patient observation." Professor **Rudolf Kalman** Univ Florida (1972 cited in [Zadeh 2011])

"Fuzziness is probability in disguise. I can design a controller with probability that could do the same thing that you could do with fuzzy logic."

Professor Myron Tribus, UCLA (Bayesian) on hearing of the fuzzy-logic control of the Sendai subway system IEEE Institute, may 1988.

And for the 90's control community in France it was even **worse**...



Well... "Fuzzy" will have 50 years in 2015

As a matter of summary... with some "fuzzyness"

Fuzzy control: the pioneers, the first applications

Limitations, getting "older"

How to go further?

Is Fuzzy dead?

Tracks

"Classical" approach of control



Goals: performances, robustness



capture



Nonlinear mapping from control to output

For example:
$$u(t) = f(\varepsilon(t), \Delta \varepsilon(t))$$

And by sake of comparison with linear control \rightarrow FC-PD





Tuning: need for optimization step or for adaptive mechanism



Ebrahim MAMDANI Imperial College of Science, Technology and Medicine, University of London (passed 2010)

Mamdani, E.H., "Advances in the linguistic synthesis of fuzzy controllers", International Journal of Man-Machine Studies, Vol. 8, pp. 669-678, 1976

1st known application:

Homblad, P. & Ostergaard, J-J. (1982) "Control of a cement kiln by fuzzy logic", in M.M. Gupta and Elie Sanchez, eds., *Fuzzy Information and Decision Processes* pp. 398-399, Amsterdam: North Holland.

Adaptive control using mechanism to modify the rule base in real-time T.J. Procyk, E.H. Mamdani "A linguistic self-organizing process controller Original" **Automatica**, Volume 15, Issue 1, January 1979, Pages 15-30

Fuzzy control history: the SOC



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Fuzzy control history: the SOC



A "simple" model for variations: mapping $\Delta y / \Delta u$ (Jacobian) SOC tested on numerous models in simulation (already reinforcement learning!)

Linear SISO Linear MIMO + delays

NL with saturations





 $\ddot{X} + 0.1X\dot{X} + 0.375X = 0.00375U$

where 0 < U < 1000 and 0 < X < 100.

Applications : Sendaï Metro (Japan) 1987



Seiji Yasunobu, Univ Tsukuba

A Fuzzy Control for Train Automatic Stop Control Seiji YASUNOBU, Shoji MIYAMOTO (Hitachi)

GOALS: security, comfort and stop precision

HITACHI Engineers: comparisons with "classical" controllers on 300,000 simulations and 3000 real-time tests

Stop precision divided by 2.5, comfort criterion x2 and consumption gain of 10%

Applications : Sendaï Metro (Japan) 1987



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Nonlinear extension of linear controllers.

Output feedback approach



- FC-P, FC-PD, FC-PI, FC-PID
- FC-adaptive

Many works comparing linear/fuzzy "like" linear [Boverie et al., 1991] [Vidolov & Melin 1993] [Pigeon 1993]

Equivalences [Siler 1989] [Ying & al. 1993] [Foulloy & Galichet 1994]

1988 Japan \rightarrow Laboratory for International Fuzzy Engineering (LIFE) 45 industrials (the only European was VW)

Thousands of applications Washing machines, cameras (Canon), vacuum cleaner (Matsushita) ... Micro-waves oven, conditioner (Mitsubishi), showers... Train, lifts, helicopter...

"counting"> 5000 [Jamshidi 1995] Recent paper about fuzzy control applications [Precup & Hellendorn 2011]

Number of fuzzy-logic-related patents issued and applied in Japan: **7,149** Number of fuzzy-logic-related patents issued in the United States: **21,878** Number of fuzzy-logic-related patents applied in the United States: **22,272**

(Data compiled by Dr. B. Tadayon, CEO, Z Advanced Computing TM, Inc.) <u>http://www.cs.berkeley.edu/~zadeh/stimfl.htm</u>

Fuzzy control: the limits

Highly complex systems and/or hard to model, identify



Stability ? Robustness?

Problem: NOT « model-based »

Fuzzy control: the limits



Input/output methods

⇒ Circle Criterion, Conicity Criterion [Ollero et al., 1993]

- linear model only: reduced interest
- PB: transform the nonlinear model: a nonlinear part + a linear part



Anibal Ollero - Professor at the Ingeniería de Sistemas y Automática Department, University of Seville, Spain

Fuzzy control: the limits



Input/Ouput Methods

$$\frac{dx}{dt} = f(\mathbf{x}) + Bu$$
$$u = \Phi(\mathbf{x})$$

Criteria based on Jacobian matrix → Necessary Conditions



As a matter of summary... with some fuzzyness

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I – Fuzzy control: going further

Brunovski form
$$\begin{cases} \dot{x}_n = f(\mathbf{x}) + g(\mathbf{x})u & \text{[Li-Xin Wang 1994]} \\ y = x_1 \\ x^T = \begin{bmatrix} x_1 & \dot{x}_1 & \dots & x_1^{(n-1)} \end{bmatrix} & u = \frac{1}{g(\mathbf{x})} \left(-f(\mathbf{x}) + y_m^{(n)} + k^T e \right) \end{cases}$$

Takes profit from universal approximation property:

$$f(\boldsymbol{x}) = \boldsymbol{w}_{f}^{T}(\boldsymbol{x})\boldsymbol{\theta}_{f}^{*} + \boldsymbol{\varepsilon}_{f}(\boldsymbol{x})$$

 $w_f(x) \rightarrow$ Fuzzy vector of basis functions

- $\theta_{f}^{*} \rightarrow$ Optimal (unknown) vector of parameters
- $\varepsilon_{f}(x) \rightarrow$ Approximation error

I – Fuzzy control: going further

Brunovski form
$$\begin{cases} \dot{x}_n = f(\mathbf{x}) + g(\mathbf{x})u & \text{[Li-Xin Wang 1994]} \\ y = x_1 \end{cases}$$
$$x^T = \begin{bmatrix} x_1 & \dot{x}_1 & \dots & x_1^{(n-1)} \end{bmatrix} \quad u = \frac{1}{g(\mathbf{x})} \left(-f(\mathbf{x}) + y_m^{(n)} + k^T e \right)$$

Indirect fuzzy adaptive control: fuzzy models used for approximating the system, i.e. $f(\mathbf{x})$ and $g(\mathbf{x})$

$$u = \frac{1}{\hat{g}(\boldsymbol{x})} \left(-\hat{f}(\boldsymbol{x}) + v + k s \right)$$

Direct fuzzy adaptive control: : fuzzy models used for approximating the control

$$u = \hat{u}(\mathbf{x})$$

II – Fuzzy control: going further

Model inversion

$$\tilde{y}_{sp_k}$$
 \tilde{M}^{-1} \tilde{u}_k \tilde{M} \tilde{y}_k

Rule by rule [Babuska et al. 1995], using optimization tools [Park et Han 2000] A more recent tentative with Piecewise Bilinear Modeling of Nonlinear Systems [Eciolaza & Sugeno 2012].

For minimum phase systems:



II – Fuzzy control: going further



Do not overestimate the solutions \rightarrow "exact" solution procedures when "possible" [Boukezzoula et al.2006], best "approximate" solution problem [Lamara et al. 2011]



III – Fuzzy control: going further



Takagi-Sugeno fuzzy models

[Takagi & Sugeno 1985] multi-model vision

Coming from "classical" fuzzy models rule based:

Rule i : If $z_1(t)$ is $F_1^i(z_1(t))$ and ... and $z_p(t)$ is $F_p^i(z_p(t))$ Then Linear model description





Takagi-Sugeno fuzzy models

[Takagi & Sugeno 1985] multi-model vision

Coming from "classical" fuzzy models rule based:

Rule i : If
$$z_1(t)$$
 is $F_1^i(z_1(t))$ and ... and $z_p(t)$ is $F_p^i(z_p(t))$
Then Linear model description
Transfert \swarrow State Space

$$y_i(t) = A_i(q)y(t) + B_i(q)u(t-nk) + \theta_{i0}$$

State Space $\begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t) \\ y(t) = C_i x(t) \end{cases}$

⇒ linear models blended together with *nonlinear functions*

 \Rightarrow Nonlinearities in the premises.





TS: sector nonlinearity approach



$$h_{i}(z(t)) \ge 0, \sum_{i=1}^{r} h_{i}(z(t)) = 1 \qquad \begin{cases} x(t+1) = \sum_{i=1}^{r} h_{i}(z(t))(A_{i}x(t) + B_{i}u(t)) \\ y(t) = \sum_{i=1}^{r} h_{i}(z(t))C_{i}x(t) \end{cases}$$

Direct from NL to TS: Sector Nonlinearity Approach [Tanaka et al. 1996]





Takagi-Sugeno fuzzy models

$$h_i(z(t)) \ge 0, \sum_{i=1}^r h_i(z(t)) = 1$$
 $x(t+1) = \sum_{i=1}^r h_i(z(t))(A_ix(t) + B_iu(t))$

Polytopic point of view

Convex hull:



Small example: $\begin{bmatrix} x_{1}(k+1) \\ x_{2}(k+1) \end{bmatrix} = \begin{bmatrix} 0.5 & 2\sin^{2}(x_{1}(k)) \\ 2-2\sin^{2}(x_{1}(k)) & 0.5 \end{bmatrix} \begin{bmatrix} x_{1}(k) \\ x_{2}(k) \end{bmatrix}$ $\sin^{2}(x_{1}(k)) \in [0,1]; \quad \sin^{2}(x_{1}(k)) = 0 \times (1-\sin^{2}(x_{1}(k))) + 1 \times \sin^{2}(x_{1}(k))$ $\begin{bmatrix} 0.5 & 2\sin^{2}(x_{1}(k)) \\ 2-2\sin^{2}(x_{1}(k)) & 0.5 \end{bmatrix} = h_{1}(x_{1}) \begin{bmatrix} 0.5 & 0 \\ 2 & 0.5 \end{bmatrix} + h_{2}(x_{1}) \begin{bmatrix} 0.5 & 2 \\ 0 & 0.5 \end{bmatrix}$

Robust stability

Discrete time LTI model $x(t+1) = A(\delta)x(t), x(0) = x_0, x(t) \in \mathbb{R}^n, A(\delta) \in \Gamma$ where Γ is an arbitrary *uncertain set*

Robust stability: the DTLTI uncertain model is said to be robustly stable if it is asymptotically stable for all $A \in \Gamma$

Problem: the set of all stable matrices **is not** a convex set

Simplest polytope:
$$A_1 - A_2$$

 $A_1 = \begin{bmatrix} 0.5 & 2 \\ 0 & 0.5 \end{bmatrix}, A_2 = A_1^T = \begin{bmatrix} 0.5 & 0 \\ 2 & 0.5 \end{bmatrix}, A = \frac{1}{2} (A_1 + A_1^T) = \begin{bmatrix} 0.5 & 1 \\ 1 & 0.5 \end{bmatrix}$
 $\lambda_i (A_1) = \lambda_i (A_2) = 0.5, i = 1, 2 \text{ and } \lambda_1 (A) = -0.5, \lambda_2 (A) = 1.5$

ATTENTION: Stability at the vertices **DO NOT** imply stability of the convex set...

Quadratic Stability of Discrete TS models

Unforced discrete TS:
$$x(t+1) = \sum_{i=1}^{r} h_i(z(t)) A_i x(t) = A_z x(t)$$

Quadratic Lyapunov function: $V(x) = x(t)^T Px(t), P = P^T > 0$

 $\Delta V(x) = x^{T}(t) \left(A_{z}^{T} P A_{z} - P\right) x(t) < 0 \underset{Complement}{\Leftrightarrow} \begin{bmatrix} -P & (*) \\ P A_{z} & -P \end{bmatrix} = \sum_{i=1}^{r} h_{i}(z(t)) \begin{bmatrix} -P & (*) \\ P A_{i} & -P \end{bmatrix} < 0$

With the convex sum property:
$$h_i(z(t)) \ge 0, \sum_{i=1}^r h_i(z(t)) = 1$$

Theorem (Tanaka and Sugeno 92): the unforced DTS is GAS if there exists a matrix *P* such that the LMI problem holds:

$$\begin{bmatrix} P & A_i^T P \\ PA_i & P \end{bmatrix} > 0, i = 1, \dots, r$$

Remark: strictly *equivalent* to quadratic stability of LTI uncertain model

Lower the conservatism and guarantee better performance / robustness



Lower the conservatism and guarantee better performance / robustness



Several ways:

- Using relaxations (Kim & Lee 01, Liu & Zhang 04, Teixeira et al. 03)

$$h_i \ge 0, i = 1, ..., r, \sum_{j=1}^r h_i = 1, \sum_{i=1}^r \sum_{j=1}^r h_i h_j \Upsilon_{ij} < 0$$

Asymptotically **closed** via Polya's theorem (Sala & Ariño 07)

Lower the conservatism and guarantee better performance / robustness



Several ways: Modifying the Lyapunov function: (Johansson et al. 99, Feng 03)

piecewise
$$V(x) = x(t)^T P_i x(t), P_i = P_i^T > 0 \quad x \in \Omega_i$$

Restriction : no improvement in the case of Sector Non Linearity approach therefore, specific TS models under consideration

Lower the conservatism and guarantee better performance / robustness



Several ways: Modifying the Lyapunov function:

Non quadratic
$$V(x) = x(t)^T \sum_{i=1}^r h_i(x) P_i x(t), P_i = P_i^T > 0$$

(Blanco et al. 01, Tanaka et al. 01, Guerra & Vermeiren 04)

Restriction : In the continuous case no real improvement excepted special case Path independent Lyapunov function (Rhee & Won 06)

A focus on the discrete case

Lower the conservatism and guarantee better performance/robustness



Modifying the Lyapunov function:

Non quadratic
$$V(x) = x(t)^T \sum_{i=1}^r h_i(x) P_i x(t), P_i = P_i^T > 0$$

$$u(x) = \sum_{i=1}^r h_i(x) F_i \left(\sum_{i=1}^r h_i(x) P_i\right)^{-1} x(t) \quad \text{(Guerra \& Vermeiren 04)}$$

Choice of the Lyapunov function



Why decreasing each step?

[Kruszewski & Guerra 05]

- Modifying the stability criteria

Classical approach







Why decreasing each step?

[Kruszewski & Guerra 05]



Example: stability

$$A_1 = \begin{bmatrix} a & -0.5 \\ 0 & -0.86 \end{bmatrix} \qquad A_2 = \begin{bmatrix} 0.87 & 0 \\ -0.5 & b \end{bmatrix}$$

The candidate Lyapunov function is: $V(x) = x^T P x$



Choice of the Lyapunov function



Choice of the Lyapunov function

Variation over k-samples

$$V(t+k)-V(t)\leq 0, k>1$$

Multiple sum extension of NQLF

$$V(x) = x^{T} \left(\sum_{i_{1}=1}^{r} \dots \sum_{i_{n}=1}^{r} h_{i_{1}}(z(t)) \dots h_{i_{n}}(z(t))^{P}_{i_{1}} x^{T} \left(\sum_{i_{1}=1}^{r} \dots \sum_{i_{n}=1}^{r} h_{i_{1}}(z(t-1)) \dots h_{i_{n}}(z(t-n)) P_{i_{1}\dots i_{n}} \right) x$$
Non Quadratic Lyapunov function (NQLF)

$$V(x) = x^{T} \left(\sum_{i_{n}=1}^{r} h_{i_{1}}(z(t)) P_{i_{1}} \right) x$$
Quadratic Lyapunov function (QLF)

$$V(x) = x^{T} \left(\sum_{i_{n}=1}^{r} h_{i_{1}}(z(t-1)) P_{i_{1}} \right) x$$

$$V(x) = x^{T} \left(\sum_{i_{n}=1}^{r} h_{i_{n}}(z(t-1)) P_{i_{n}} \right) x$$

$$V(x) = x^{T} \left(\sum_{i_{n}=1}^{r} h_{i_{n}}(z(t-1)) P_{i_{n}} \right) x$$

New Observer design : Results

Example: Consider the following TS model with *a* and *b* are free parameters.

$$A_{1} = \begin{bmatrix} 2.5 & 1 \\ 0.5 & 2 \end{bmatrix} \quad A_{2} = \begin{bmatrix} 0.5 & 0 \\ 2.5 & a \end{bmatrix} \quad B_{1} = B_{2} = \begin{bmatrix} 1 & 0 \end{bmatrix}^{T} \quad C_{1} = \begin{bmatrix} b & 1 \end{bmatrix} \quad C_{2} = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

[Kerkeni 2011] IC engine cylinder pressure estimation



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Cylinder by cylinder estimation

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Periodic TS model

Periodic TS observer

$$\begin{cases} x(t+1) = A_{z(t)}^{(c)} x(t) + B_{z(t)}^{(c)} u(t) \\ y(t) = C_{z(t)}^{(c)} x(t) \end{cases} \qquad c = t \mod \\ \begin{cases} \hat{x}(t+1) = A_{z(t)}^{(c)} \hat{x}(t) + B_{z(t)}^{(c)} u(t) + S_{z(t)}^{(c)^{-1}} K_{z(t)}^{(c)} (y(t) - \hat{y}(t)) \\ \hat{y}(t) = C_{z(t)}^{(c)} \hat{x}(t) \end{cases}$$
For each set is a set in the set of the set o

Structure of LMI constraint

$$S_{j}^{(m)} \in \mathbb{R}^{7 \times 7} \qquad \qquad \Omega_{ij}^{m} = \begin{bmatrix} -\tilde{P}^{(m)} & (*) \\ S_{j}^{(m+1)} A_{i}^{(m)} - K_{j}^{(m)} C_{i}^{(m)} & -S_{j}^{(m)} - \left(S_{j}^{(m)}\right)^{T} + \tilde{P}^{(m)} \end{bmatrix}$$

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Fuzzy applications → plenty

Robotics and control, a very close history



Trajectory planning Obstacle avoidance







Cooperative robotics



language for prog humanoid robot





TOKYO UNIVERSITY OF SCIENCE

As a matter of summary... with some fuzzyness

Fuzzy control: the pioneers, the first applications

Limitations, getting "older"

How to go further?

Is Fuzzy dead?

Tracks

Fuzzy control: the community

Annual international conference Fuzz'IEEE / WCCI (~200 attendants) International Journals IEEE Trans. On Fuzzy Systems (2012 IF 4,26) Fuzzy Sets and Systems (2012 IF 1,8)

IFAC Technical Committee 3.2 (Chair Antonio Ruano) IFAC ICONS triennial

But also any IFAC Journal, conf + IEEE SMC, TAC ...



Prof. Antonio Sala (PU Valencia)



Prof. FENG, Gang (Gary) Hong Kong



Jong-Hwan Kim KAIST (Seoul)



Prof. Robert Babuska (TU Delft)





Some tracks...

Type II Fuzzy sets - Jerry Mendel, Pr Univ South California

Mixing with reinforcement learning Franck Lewis Pr Univ Texas at Arlington

Problems:

- high sized systems
- output feedback with performances
- networked systems

Any way to come back to Human? Man in the loop? Is there any place/need for fuzzy mathematical tools (uncertainty, linguistic...)

We need new tools, new ideas, new gaps





Future events of TC 3.2



http://cescit2015.um.si/ Deadline for receipt of papers: 20 October, 2014

4th IFAC Conference on Intelligent Control and Automation Science -ICONS 2016

1 proposition in Reims, France (K. Guelton)



"Only those who attempt the absurd can achieve the impossible." Maurits Cornelis Escher





"If we knew what it was we were doing, it would not be called research, would it?"

"The truth is rarely pure and never simple." Oscar Wilde



Thank you